

1. (10 points) The velocity of a particle is given by the equation $v(t) = 3t^2 - 6t + 1$ ft/s, where t is in seconds. Determine the position and acceleration of the particle when t is equal to 2 seconds.

$$v = 3t^2 - 6t + 1$$

$$\frac{ds}{dt} = 3t^2 - 6t + 1$$

$$s = t^3 - 3t^2 + t$$

$$s = (2)^3 - 3(2)^2 + (2)$$

$$s = 22.0 \text{ ft}$$

$$v = 3t^2 - 6t + 1$$

$$\frac{dv}{dt} = 6t - 6$$

$$a = 6t - 6$$

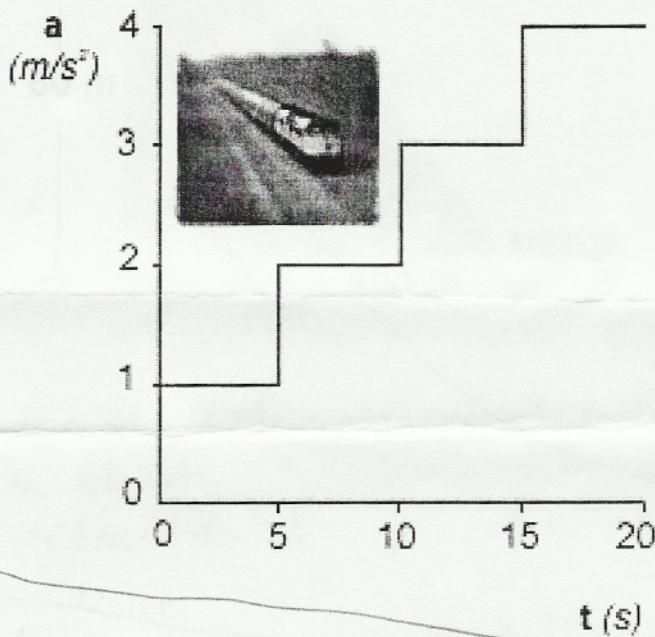
$$a = 6(2) - 6$$

$$a = 6.00 \text{ ft/s}^2$$

(5)

2. (30 points) You are a summer student at VIA Rail, where the engineers have proposed an incremental acceleration scheme, shown below in the $a-t$ plot. On your first day you are assigned the following duties:

- construct the $v-t$ plot showing the velocity at 5, 10, 15, and 20 seconds,
- construct the $s-t$ plot showing the position at 5, 10, 15, and 20 seconds,
- convert the final velocity and position into km/h and km respectively, for the VIA marketing staff.



Given

$$a(0 \leq t \leq 5) = 1 \text{ m/s}^2$$

$$a(5 \leq t \leq 10) = 2 \text{ m/s}^2$$

$$a(10 \leq t \leq 15) = 3 \text{ m/s}^2$$

$$a(15 \leq t \leq 20) = 4 \text{ m/s}^2$$

Find \rightarrow A) $v-t$ plot

B) $s-t$ plot

C) Final velocity in km/h and position in km .

Solution

$$A) 0 \leq t \leq 5$$

$$a = 1$$

$$v = t + C$$

$$v = t$$

$$5 \leq t \leq 10$$

$$a = 2$$

$$v = 2t + C$$

$$v = 2t - 5$$

$$10 \leq t \leq 15$$

$$a = 3$$

$$v = 3t + C$$

$$v = 3t - 15$$

$$15 \leq t \leq 20$$

$$a = 4$$

$$v = 4t + C$$

$$v = 4t - 30$$

(30)

$$v(t) = t$$

$$v(5) = 5.00 \text{ m/s}$$

$$v(t) = 2t - 5$$

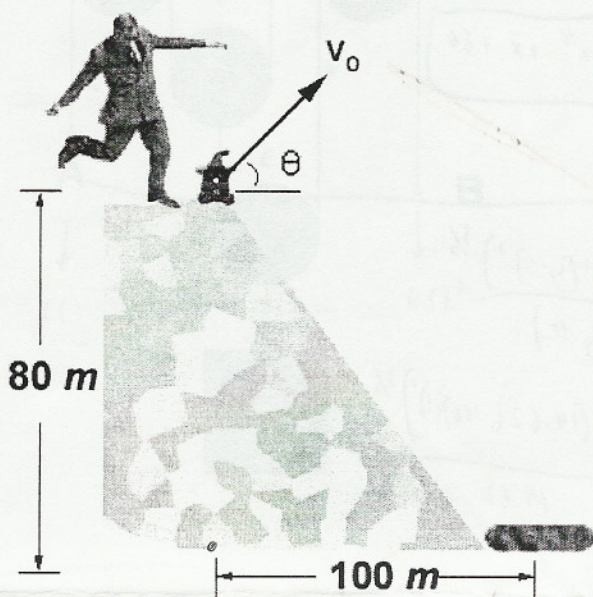
$$v(10) = 15.0 \text{ m/s}$$

3. (30 points) Given the hypothetical situation shown in the figure below, determine the following:

- What speed ($|v_0|$) is required to project the Furby to the safety mat?
- What is the equation of the Furby's path ($y = f(x)$)?
- What are the normal and tangential components of acceleration just prior to the furby's landing?

Assume that the launch angle (θ) is 45° relative to the horizon, and that the furby can be accurately modeled as projectile.

Editors Note: Furbies are not animals; rather, they are toys that talk. The College of Engineering does not condone violence towards animals.



Given

$$x_0 = 0 \text{ m}$$

$$y_0 = 80 \text{ m}$$

$$x = 100 \text{ m}$$

$$y = 0 \text{ m}$$

Find \rightarrow A) V_0

B) equation

C) a_t and a_n

tion

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$x = x_0 + v_0 (\cos \theta) t$$

$$100 = v_0 (\cos 45^\circ) t$$

$$100 = 0.707 v_0 t$$

$$t = \frac{100}{0.707 v_0}$$

$$t = \frac{141.4}{v_0}$$

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$0 = 80 + v_0 (\sin \theta) t + \frac{1}{2} a t^2$$

$$0 = 80 + (\sin 45^\circ) (141.4) + \frac{1}{2} a \left[\frac{141.4^2}{v_0^2} \right]$$

$$0 = 80 + 100 + \frac{98070}{v_0^2}$$

$$-180 = \frac{98070}{v_0^2}$$

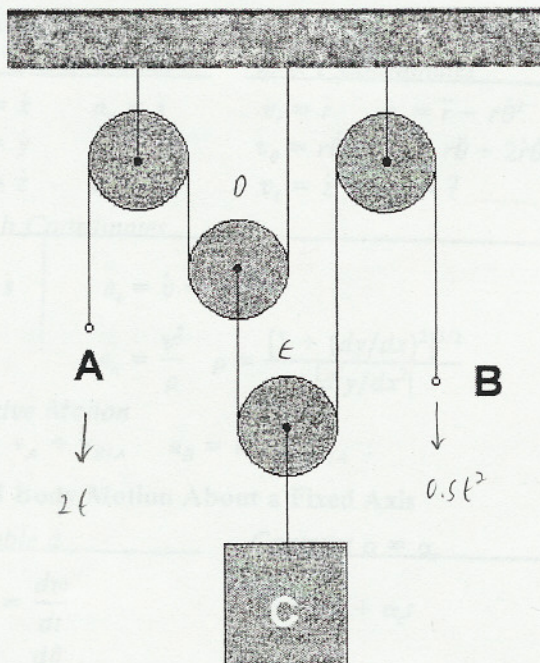
$$v_0^2 + a (s - s_0)$$

$$v_0^2 (\sin^2 \theta) + 9.81 (0 - 80)$$

13

4. (30 points) For the pulley arrangement shown in the figure below, the velocity of points A and B are $2t$ (ft/s) and $0.5t^2$ (ft/s) in the downward direction, respectively. Determine:

- the velocity of block C as a function of time,
- the relative velocity of C with respect to A as a function of time,
- the relative acceleration of C with respect to A when $t = 1$ s.



Given

$$v_A = 2t$$

$$v_B = 0.5t^2$$

Find \rightarrow A) v_C

B) v_{CA}

C) a_{CA}

18/30

$$A) \quad l = s_A + 2s_D$$

$$0 = v_A + 2v_D$$

$$v_D = -\frac{1}{2}v_A$$

$$v_D = -\frac{1}{2}(2t)$$

$$v_D = -t$$

$$l = s_B + 2s_E + s_D$$

$$0 = v_B + 2v_E + v_D$$

$$v_E = \frac{1}{2}(-v_B - v_D)$$

$$v_E = \frac{1}{2}(-0.5t^2 - t)$$

$$v_E = -\frac{1}{4}t^2 - \frac{1}{2}t$$

$$v_C = v_E$$

$$v_C = -\frac{1}{4}t^2 - \frac{1}{2}t$$

direction?

Carried over

6

$$B) \quad v_{CA} = v_C - v_A$$

$$v_{CA} = (-\frac{1}{4}t^2 - \frac{1}{2}t) - (2t)$$

$$v_{CA} = -\frac{1}{4}t^2 - \frac{5}{2}t$$

$$C) \quad v_A = 2t$$

$$a_A = 2$$

$$v_C = -\frac{1}{4}t^2 - \frac{1}{2}t$$

$$a_C = -\frac{1}{2}t - \frac{1}{2}$$